



Application of the transfer configuration factors in radiation heat transfer

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Abstract

A new engineering method is presented for the analysis of radiation exchange within an open enclosure filled with an isothermal radiation gas. It is shown that the general class can be formulated of an open enclosure composed of black, diffusive-grey or spectral quantity. A number of new relations are derived between temperature, heat flux and effective radiation energy flux of surfaces. The method of determination of transfer configuration factors and dissipation losses is also presented. Whereas in previously-published methods, errors were possible if an equilibrium state were assumed, the current approach does not have this drawback, but enables the radiation gas temperature to be explicitly predicted. A special emphasis is put on the problems of radiation in enclosures composed of adiabatic surfaces which affect the measurement technique of surface emissivity. Calorimetric emission measurements of thermal radiation properties of opaque surfaces are also presented. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

a_{kj} matrix elements defined by equation (26)
 A area
 \mathbf{A} matrix \mathbf{A} defined by equation (29)
 A_{kj} inverse matrix elements
 \mathbf{B} matrix \mathbf{B} defined by equation (30)
 \mathbf{C} matrix \mathbf{C} defined by equation (31)
 C_k matrix elements defined by equation (27)
 e emissive power (emission flux)
 $e_{lb,k}$ band emissive power
 e_{zb} hemispherical emissive power
 F configuration factor or abbreviation defined by equation (69)
 \bar{F}, \bar{F}_l transfer and band transfer configuration factors
 j, k indexes denoting individual surfaces
 l band
 M_n, M_{ln}, M abbreviation quantity defined by equations (13), (42) or (61)–(63)
 n total number of surfaces (real and semi-transparent membranes)
 N number of real surfaces in a spatial system
 q energy flux
 Δq_l band energy
 $Q, Q_{s,p}$ energy per unit time and dissipation loss
 R abbreviation defined by equation (67)

T absolute temperature
 Z matrix Z defined by equations (78)–(80).

Greek symbols

δ Kronecker delta
 ε emissivity
 ε_l selective emission coefficient
 $\varepsilon_{g,k-j}$ emittance of radiation gas
 $\varepsilon_{l,k-j}$ band emittance of radiation gas
 λ wavelength
 ν wavenumber
 ρ reflectivity
 σ Stefan–Boltzmann constant
 τ_{k-j} transmission factor (transmittance)
 $\tau_{l,k-j}$ band transmittance
 ψ_c abbreviation defined by equation (68).

Subscripts

a adiabatic surface
 b blackbody or dissipation gas
 g radiation gas
 i incident flux
 j, k, z property of surface j, k or z
 $j-k$ direction from surface j to surface k
 0 effective flux

- s scattering loss
 α, β property of semi-transparent surface
 1, 2 surface 1 or 2.

Superscript

- * index of an incident heat flux in radiation gas.

1. Introduction

Modern engineering technology is replete with systems in which thermal radiation is a dominant mode of energy transfer. High-temperature heat transfer devices are often associated with large convective and radiant heat fluxes and the radiant flux becomes of greater importance as the temperature level increases. The net effect of this exchange is to transfer heat from high temperature regions to those at the lower temperatures. There is also increasing interest in gaseous radiative heat transfer. The usual exact formulation of the radiation interchange within an enclosure containing an absorbing-emitting non-isothermal gas by the use of integral equations becomes so complex even after simplifying assumptions such as greyness of gases and surfaces, that a solution becomes prohibitively tedious. Previously-published treatments have not included consideration of an active enclosure in which radiation passed through windows or semi-transparent membranes. An alternative approach is made here to derive a new fast method of engineering usefulness. It provides a set of analytical tools for treating such enclosures (with windows or semitransparent membranes) and obtaining the radiative behaviour of radiation gas contained within them. The present paper also provides the additional analysis necessary to extend refs. [2, 4, 10–12, 14] to include a radiative contribution of a radiation gas in an open enclosure. The main objective of this paper is to introduce a new transfer configuration factor \bar{F} in practical, engineering, radiation heat transfer contexts. The well-known net-radiation method of Hottel [14] Poljak [1, 12] and Gebhart [7] generalized here to include multidimensional configuration enclosures which may be open or closed in character, and filled with an isothermal radiation grey gas or gas of known selective emissivity. In addition, two examples demonstrate the reliability and accuracy of the theoretical analysis and its agreement with the First and Second Laws of Thermodynamics. The method is illustrated by results of some experimental measurements of emissivity on opaque surfaces. The present analysis is also a development of that presented by Siegel and Howell [14] (it has deliberately used the same notation and derivation, to enable the reader to appreciate the additional features of this work). However, in the present paper the same general model is retained but the analysis is extended to include the thermal interaction of the external surface of the two- and four-surface open enclosure with various kinds of con-

ditions. In this sense an open enclosure is treated as a system of n -surfaces exchanging thermal radiation internally while simultaneously experiencing radiant heat exchange with ambient.

2. Radiation between finite diffuse grey areas of an open enclosure filled with isothermal radiation grey gas

From the enclosure theory [1, 11, 12, 14] it is evident that the net radiation method is a powerful analytical tool that can, in many situations, be much less difficult to apply than the ray-tracing method. The net-radiation method [12] is applied here to derive the radiation characteristics of an open enclosure filled with isothermal radiation grey gas. The theoretical analysis is based on the following assumptions [2, 15]:

- the open system is in thermodynamic equilibrium,
- the surfaces under consideration are homogeneous, grey and defined by physical properties (namely ϵ and ρ_λ) independent of wavelength λ ,
- the radiation gas inside the enclosure is chemically homogeneous and isothermal.

2.1. Effective and incident radiation energy flux

Consider an open enclosure of N discrete surfaces each at a uniform temperature, and α, β, \dots, r semitransparent membranes (abstract surfaces), as shown in Fig. 1. Some of the surfaces can be open boundaries such as windows or holes. These are usually modelled as perfectly black surfaces either at the temperatures of the respective parts of the surroundings outside the openings (windows) or at zero absolute (holes). The open enclosure is filled with an absorbing-emitting medium at a predetermined, known emissivity and uniform temperature T_g [2, 15].

The net-radiation method of Poljak [12] is extended here to include terms for a radiation medium (radiation

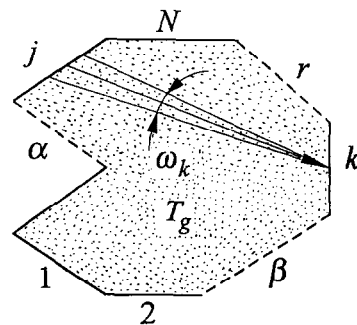


Fig. 1. An open enclosure composed of N discrete surface areas and α, β, \dots, r semitransparent membranes and filled with uniform an absorbing-emitting medium g (shown in cross section for simplicity).

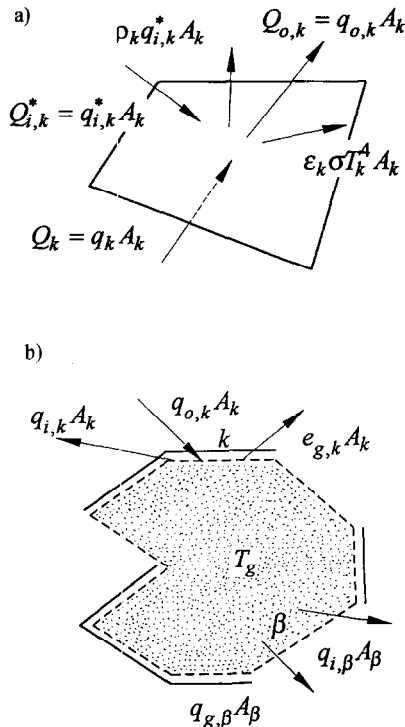


Fig. 2. Geometry for derivation of the engineering treatment of radiation. (a) Energy quantities incident on and leaving typical surface area of an open enclosure. (b) Energy quantities incident on and leaving of an absorbing-emitting medium on examples of surface A_k and semitransparent membrane A_β .

gas). At the k -th surface of an open enclosure, as shown in Fig. 2(a), a heat balance gives:

$$Q_k = q_k A_k = (q_{o,k} - q_{i,k}^*) A_k. \quad (1)$$

The $q_{o,k}$ and $q_{i,k}^*$ are, respectively, the effective and incident radiation energy fluxes, and Q_k is the external energy supplied to A_k . The effective radiation energy flux is composed of emitted and reflected energy [14, 15] as given by:

$$Q_{o,k} = (e_k + \rho_k q_{i,k}^*) A_k = [e_k + (1 - \epsilon_k) q_{i,k}^*] A_k. \quad (2)$$

In equation (2) e_k is the emissive power of surface A_k at T_k . The $q_{i,k}^*$ in equation (1) is the incident radiation energy flux to A_k . It is equal to the sum of the contributions from all surfaces (alien and own) that reach the k -th surface after allowance for (i) absorption in passing through the intervening radiation gas plus (ii) contribution due to emission from the gas:

$$q_{i,k}^* = q_{i,k} + e_{g,k}. \quad (3)$$

Using the definitions introduced in [14–16], the incident radiative energy flux, for a typical pair of surfaces j – k , can be written as:

$$q_{i,k} = \sum_{j=1}^N F_{k-j} \tau_{k-j} q_{o,j}. \quad (4)$$

The second term in equation (3) is the incident energy received at A_k as a result of emission (via the radiation gas) from n surfaces bounding this isothermal gas at T_g of uniform composition and density [15]:

$$e_{g,k} = e_{g,b} \sum_{j=1}^n \epsilon_{g,k-j} F_{k-j} \quad (5)$$

where F_{k-j} , τ_{k-j} and $\epsilon_{g,k-j}$ are the geometrical-mean configuration factor, transmittance and emittance of gas, respectively.

Equations (1)–(3) form set of relationships between $q_{o,k}$ and $q_{i,k}^*$ and q_k for each of the surfaces in the open enclosure to the e_b for that surface and to $e_{g,b}$. By substituting equations (5) and (4) into equation (3) and then equation (3) into equation (2) we obtain, respectively:

$$q_{i,k}^* = \sum_{j=1}^N F_{k-j} \tau_{k-j} q_{o,j} + e_{g,b} \sum_{j=1}^n \epsilon_{g,k-j} F_{k-j} \quad (6)$$

and

$$q_{o,k} = \epsilon_k \sigma T_k^4 + (1 - \epsilon_k) q_{i,k}^*. \quad (7)$$

The first term of equation (6) is the radiation energy leaving A_j that is transmitted to A_k . The second term is the radiation energy received at A_k as a result of emission by the constant-temperature radiation gas in the envelope between A_j and A_k . This envelope is the volume occupied by all straight paths between any part of A_j and A_k .

2.2. Heat balance of gas–gas emission flux

In order to calculate effective and incident energy flux at surface A_k the gas temperature and hence gas emission flux $e_{g,b}$ both have to be found or specified. The gas emission flux $e_{g,b}$ can be determined from a heat balance on the gas. Figure 2(b) shows a schematic diagram of gas surrounded by a balance shield on which are marked positions of incoming and outgoing energy flows on examples of a surface A_k and semitransparent membrane A_β . A heat balance on the gas provides a relation between the gas temperature and the energy supplied to the gas and can be written as:

$$\sum_{k=1}^N A_k q_{o,k} = \sum_{k=1}^N A_k q_{i,k} + \sum_{r=2}^{r=\beta, \gamma, \dots} A_r q_{i,r} + \sum_{k=1}^n A_k e_{g,k}. \quad (8)$$

In equation (8) the third component is the sum of energy leaving through a semitransparent membrane outside an open-enclosure—it is the sum of radiation from all the real surfaces N . For example the total energy escaping at a membrane A_r , is:

$$A_r q_{i,r} = A_1 \tau_{1-r} F_{1-r} q_{o,1} + A_2 \tau_{2-r} F_{2-r} q_{o,2} + \dots + A_k \tau_{k-r} F_{k-r} q_{o,k} + \dots + A_N \tau_{N-r} F_{N-r} q_{o,N}. \quad (9)$$

Then for an arbitrary geometry of open enclosure and radiation gas equation (8) becomes:

$$\sum_{k=1}^N A_k q_{0,k} = \sum_{k=1}^N A_k e_{g,k} + \sum_{j=1}^N A_j q_{0,j} \sum_{z=1}^N F_{j-z} \tau_{j-z} \quad (10)$$

Finally, by using the reciprocal and configuration factor relations for enclosures, equation (10) can then be written as:

$$\sum_{j=1}^N A_j q_{0,j} \sum_{z=1}^N F_{j-z} \varepsilon_{g,j-z} = \sum_{k=1}^N A_k e_{g,k} \quad (11)$$

This can be evaluated by inserting equation (5) into equation (11). This results in the relationship:

$$e_{g,b} = \frac{\sum_{j=1}^N A_j q_{0,j} \sum_{z=1}^N F_{j-z} \varepsilon_{g,j-z}}{M_n} \quad (12)$$

where

$$M_n = \sum_{k=1}^N A_k \sum_{j=1}^N \varepsilon_{g,k-j} F_{k-j} \quad (13)$$

Now by substituting equation (12) into the equations (6) and (7), this pair can be written in the following form [3, 15]:

$$q_{i,k}^* = \sum_{j=1}^N F_{k-j} \tau_{k-j} q_{0,j} + \frac{\sum_{j=1}^N A_j q_{0,j} \sum_{z=1}^N F_{j-z} \varepsilon_{g,j-z}}{M_n} \sum_{j=1}^n \varepsilon_{g,k-j} F_{k-j} \quad (14)$$

$$q_{0,k} = \varepsilon_k \sigma T_k^4 + (1 - \varepsilon_k) q_{i,k}^* \quad (15)$$

In equations (14) and (15) the gas temperature T_g has been eliminated as it is the function of optical properties of the whole system and boundary conditions. Usually a gas temperature is guessed and $e_{g,b}$ is found. The solution is repeated for several gas temperatures and the resulting relation between $e_{g,b}$ and T_g yields the T_g corresponding to the specified $e_{g,b}$ by equation (12) [14, 15]

2.3. Definition of transfer configuration factor

Equations (1), (2) and (4) are simultaneous relations between q_k , T_k , $q_{0,k}$ and $q_{i,k}^*$ for each surface and through (4), to the other surfaces. By introducing two formulae for $q_{i,k}^*$ [equation (7) and equation (14)] into the energy balance (1) these provide two heat balance equations for Q_k in terms of T_k , $q_{0,k}$ and the optical properties of the open enclosure:

$$Q_k = A_k \left\{ q_{0,k} - \left[\sum_{j=1}^N F_{k-j} \tau_{k-j} q_{0,j} + \frac{\sum_{j=1}^N A_j q_{0,j} \sum_{z=1}^N F_{j-z} \varepsilon_{g,j-z}}{M_n} \sum_{j=1}^n \varepsilon_{g,k-j} F_{k-j} \right] \right\} \quad (16)$$

$$Q_k = A_k \frac{\varepsilon_k}{1 - \varepsilon_k} (\sigma T_4^k - q_{0,k}) \quad (17)$$

The set of equations (16) and (17) can be further reduced by solving (17) for $q_{0,k}$ and inserting it into (16). This results in the relation [15]:

$$q_k = \frac{Q_k}{A_k} = \sum_{j=1}^N (\delta_{kj} - \bar{F}_{k-j}) q_{0,j} \quad (18)$$

where $k = 1, 2, 3, \dots, N$, $\delta_{kj} =$ Kronecker delta, $\delta_{kj} = \begin{cases} 1 & \text{when } k=j \\ 0 & \text{when } k \neq j \end{cases}$, $\bar{F}_{k-j} =$ transfer configuration factor.

\bar{F}_{k-j} fulfils the relationship of the so-called generalised principle of unity and reciprocity [2, 15],

$$\sum_{j=1}^N \bar{F}_{k-j} = 1; \quad A_k \bar{F}_{k-j} = A_j \bar{F}_{j-k} \quad (19)$$

and thus, the following generalized form for an open enclosure filled with an isothermal radiation gas

$$\bar{F}_{k-j} = F_{k-j} \tau_{k-j} + \frac{A_j \sum_{z=1}^n F_{j-z} \varepsilon_{g,j-z}}{M_n} \sum_{j=1}^n F_{k-j} \varepsilon_{g,k-j} \quad (20)$$

For a nonabsorbing medium $\varepsilon_{g,k-j} = 0$; $\tau_{k-j} = 1$ and equation (20) becomes the well known configuration factor dependent only on geometry.

As a first approach to understanding the application of the analysis, consider that equations (17) and (18) can be written for each of the N surfaces in the open enclosure. This provides $2N$ equations for $2N$ unknowns, if, for example, T_g is specified. The q_0 will form N of these unknowns. The remaining unknowns will consist of q and T depending on what boundary quantities are specified.

2.4. Relations between surface temperature T and surface heating q

The relation between surface temperature and surface incoming heat flux is obtained from equations (17) and (18) by elimination of effective energy flux q_0 [14, 15]:

$$\sum_{j=1}^N \left(\frac{\delta_{kj}}{\varepsilon_j} - \bar{F}_{k-j} \frac{1 - \varepsilon_j}{\varepsilon_j} \right) q_j = \sum_{j=1}^N (\delta_{kj} - \bar{F}_{k-j}) \sigma T_j^4 \quad (21)$$

If equation (21) is written for each k from 1 to N , a set of N equations is obtained relating the $2N$ quantities q_j and T_j for the surface. Assuming the radiation gas temperature or its properties are known, then one-half of the q_j and T_j values need to be specified, and the equations can be solved for the remaining unknowns. The situation becomes more complicated if some of the unknowns refer to the temperature dependent properties of the radiation gas and surfaces.

In this case the set of equations can be solved by the method of successive approximations until the value of the surface emission ε and T or \bar{F} becomes constant in the following iterations.

2.5. Solution method in terms of effective radiation energy flux

An alternative approach for computing radiative exchange is to solve for q_0 for each surface and then compute the q , T and radiation gas temperature T_g . Of course, in the previous formulation the q_0 can be found from the q and T by using equation (17).

When the surface temperatures are all specified, the set of simultaneous equations for q_0 is obtained by eliminating Q_k from equations (17) and (18). This yields the following equations for the k -th surface :

$$\sum_{j=1}^N [\delta_{kj} - (1 - \epsilon_k) \bar{F}_{k-j}] q_{0,j} = \epsilon_k \sigma T_k^4 \tag{22}$$

With the T given, the $q_{0,j}$ can be found. Then if desired equation (17) can be used to compute q for each surface. If however q is specified for some surfaces and T for others, equation (22) is used for the surfaces with known T in conjunction with equation (18) for the surfaces with known q to obtain the set of simultaneous equations for the unknown q_0 . In a general form, if an open enclosure has surfaces $1, 2, \dots, m$ with given temperatures and the remaining surfaces $m + 1, m + 2, \dots, N$ with specified heat input, then the system of equations for the q_0 is, from equations (22) and (18),

$$\sum_{j=1}^N [\delta_{kj} - (1 - \epsilon_k) \bar{F}_{k-j}] q_{0,j} = \epsilon_k \sigma T_k^4 \quad 1 \leq k \leq m \tag{23}$$

and

$$\sum_{j=1}^N (\delta_{kj} - \bar{F}_{k-j}) q_{0,j} = q_k \quad m + 1 \leq k \leq N. \tag{24}$$

Equations (23) and (24) presented above allow the calculation of most engineering cases.

2.6. Matrix inversion for open enclosure equations

A set of equations such as (23) and (24) can be written in a shorter form. Let the right hand side be C_k and the quantities in parentheses on the left be a_{kj} . Then the set of k equations can be written as :

$$\sum_{j=1}^N a_{kj} q_{0,j} = C_k \tag{25}$$

where

$$a_{kj} = \begin{cases} \delta_{kj} - (1 - \epsilon_k) \bar{F}_{k-j} & 1 \leq k \leq m \\ \delta_{kj} - \bar{F}_{k-j} & m + 1 \leq k \leq N \end{cases} \tag{26}$$

$$C_k = \begin{cases} \epsilon_k \sigma T_k^4 & 1 \leq k \leq m \\ q_k = \frac{Q_k}{A_k} & m + 1 \leq k \leq N \end{cases} \tag{27}$$

Equation (25) can also be written with the use of matrix in the following form :

$$[\mathbf{A}] \cdot [\mathbf{B}] = [\mathbf{C}] \tag{28}$$

The array of a_{kj} coefficient is the matrix of coefficients and is designated by a bracket notation

$$[\mathbf{A}] \equiv [a_{kj}] \equiv \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1j} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2j} & \dots & a_{2N} \\ - & - & - & - & - & - \\ a_{k1} & a_{k2} & \dots & a_{kj} & \dots & a_{kN} \\ - & - & - & - & - & - \\ a_{N1} & a_{N2} & \dots & a_{Nj} & \dots & a_{NN} \end{bmatrix} \tag{29}$$

$$[\mathbf{B}] \equiv [q_{0,j}] \equiv \begin{bmatrix} q_{0,1} \\ q_{0,2} \\ \dots \\ q_{0,j} \\ \dots \\ q_{0,N} \end{bmatrix} \tag{30}$$

$$[\mathbf{C}] \equiv [C_k] \equiv \begin{bmatrix} C_1 \\ C_2 \\ \dots \\ C_j \\ \dots \\ C_N \end{bmatrix} \tag{31}$$

and the elements of the matrix $\mathbf{C} \equiv [C_k]$ are defined by the following equation

$$C_k = a_{k1} q_{01} + a_{k2} q_{02} + \dots + a_{kj} q_{0j} + \dots + a_{kN} q_{0N}. \tag{32}$$

A method of solving a set of equations such as (32) is to obtain a second matrix $[\mathbf{A}]^{-1}$ (that is the inverse of the matrix $[\mathbf{A}]$)

$$[\mathbf{B}] = [\mathbf{A}]^{-1} [\mathbf{C}]. \tag{33}$$

The inverse matrix has terms A_{kj} corresponding to each a_{kj} in the original matrix [14]. After the inverse matrix has been obtained, the $q_{0,k}$ values in equations (23) or (24) are found as the sum of products of A_{kj} and C_k .

$$q_{0,k} = \sum_{j=1}^N A_{kj} C_j. \tag{34}$$

Thus, the solution for each $q_{0,k}$ is in the form of a weighted sum $\epsilon_k \sigma T_k^4$ and q_k multiplied, respectively, by the coefficient of the inverse matrix A_{kj} . For a given open enclosure the configuration factors \bar{F}_{k-j} remain fixed. If in addition the ϵ_k and radiation gas properties are constant, then the elements a_{kj} and hence the inverse elements A_{kj} , remain fixed for the open enclosure. The fact that the A_{kj} 's remain fixed has utility when it is desired to compute the radiation qualities within an open enclosure for many different boundary conditions (different values of T and q_k) [11, 14, 15].

2.7. Dissipation loss of an open enclosure

As already mentioned in the Introduction, there may be semitransparent membranes in an open enclosure and

part of the radiative energy from the surfaces and the radiation gas leaves the system through them without returning. This energy is called the ‘dissipation loss’ Q_s . By considering an open enclosure formed by two infinite surfaces and two semitransparent membranes as shown in Fig. 3(a), Q_s can be determined from equation (14) as:

$$Q_s = Q_{0,1}(1 - \bar{F}_{1-2} - \bar{F}_{1-1}) + Q_{0,2}(1 - \bar{F}_{2-1} - \bar{F}_{2-2}). \quad (35)$$

Equation (35) also can be transformed for any open system to the following:

$$Q_s = \sum_{j=1}^N Q_{0,j} \left(1 - \sum_{k=1}^N \bar{F}_{j-k} \right). \quad (36)$$

The result is a general relation and in the case of a closed enclosure equals zero [2, 15].

3. Band radiation of isothermal gases within a nongrey open enclosure

The situation considered here is the radiation heat exchange in an open enclosure with selective properties

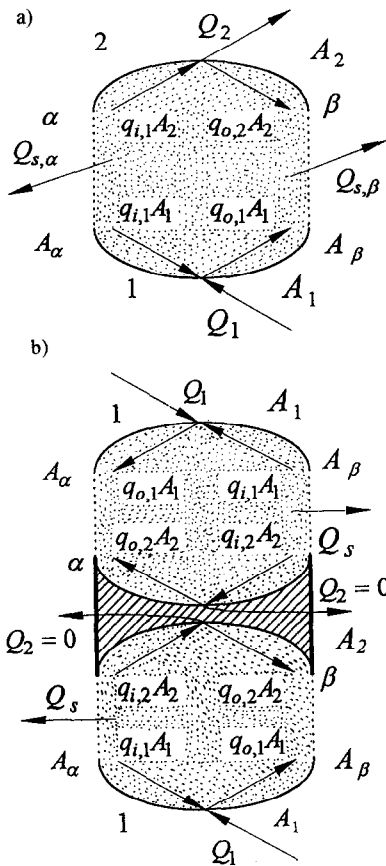


Fig. 3. Geometry of an open enclosure. (a) Two surface open enclosure. (b) The open system composed of two simple two surface open enclosure.

depending on the wavelength λ . By the integration of these properties with respect to λ from $0-\infty$, the total quantities can be obtained [14, 15]. For the parallel plate case only analytical or numerical integration can be done rather easily. It is however almost impossible with many surfaces and for a gas having irregular and spectrally-dependent properties. Some reduction in accuracy of the integration may be reasonable in practical applications, because of the uncertainty already present in many of the spectral property values that are used. The band energy approximation is the term used to designate the conceptually simple method of replacing each single integral extending over all wavelengths by a summation of smaller integrals that each extend over a portion of a spectrum in which averaged values are used.

3.1. Equations of band radiation heat transfer

In the following section an approach is developed for integrating over the wavelength by dividing the spectrum into bands in which the radiation gas is either absorbing or non-absorbing. A detailed development is given by Nelson [10] of the band relations for a nongrey isothermal gas in an enclosure with diffuse walls. For a more complex open enclosure with diffuse walls, the solution of equation (21) in matrix form can be obtained as:

$$\sum_{j=1}^N \left(\frac{\delta_{kj}}{\epsilon_{lj}} - \bar{F}_{l,k-j} \frac{1 - \epsilon_{lj}}{\epsilon_{lj}} \right) \Delta q_{lj} = \sum_{j=1}^N (\delta_{kj} - \bar{F}_{l,k-j}) \epsilon_{l,h,j} \Delta \lambda_l. \quad (37)$$

In equation (37) the index l takes one of the values within the range $1, 2, \dots, L$, and refers to an individual band of width $\Delta \lambda_l$. For the same conditions equations (17), (18) and (22) yields:

$$\Delta q_{l,k} = \frac{\epsilon_{l,k}}{1 - \epsilon_{l,k}} (e_{l,b,k} \Delta \lambda_l - \Delta q_{0l,k}) \quad (38)$$

$$\Delta q_{l,k} = \sum_{j=1}^N (\delta_{kl} - \bar{F}_{l,k-j}) \Delta q_{0l,j} \quad (39)$$

$$\sum_{j=1}^N [\delta_{kj} - (1 - \epsilon_{l,k}) \bar{F}_{l,k-j}] \Delta q_{0l,j} = \epsilon_{l,k} e_{l,b,k} \Delta \lambda_l \quad (40)$$

where the transfer band configuration factor $\bar{F}_{l,k-j}$ is given by

$$\bar{F}_{l,k-j} = F_{k-j} \tau_{l,k-j} + \frac{A_j \sum_{z=1}^n F_{j-z} \epsilon_{l,j-z}}{M_{l,n}} \sum_{j=1}^n F_{k-j} \epsilon_{l,k-j} \quad (41)$$

and

$$M_{l,n} = \sum_{k=1}^n A_k \sum_{j=1}^n \epsilon_{l,k-j} F_{k-j}. \quad (42)$$

3.2. Band radiation for given temperature of all surfaces

After the $M_{l,n}$ factor has been obtained, the effective radiation energy flux Δq_{0l} is calculated from equation

(40) then using equation (39) the values of supplied heat flux $\Delta q_{l,k}$ may be found. In matrix form equation (40) can be written as:

$$\mathbf{A}_l \cdot \mathbf{B}_l = \mathbf{C}_l \tag{43}$$

Thus, the effective radiation band energy can be calculated as:

$$\mathbf{B}_l = \mathbf{A}_l^{-1} \mathbf{C}_l \tag{44}$$

or

$$\Delta q_{0l,k} = \sum_{j=1}^n A_{kj}(l) C_{lj} \tag{45}$$

where $A_{kj}(l)$ are the elements of the inverse matrix \mathbf{A}_l^{-1} and $\det \mathbf{A}_l \neq 0$.

After the band effective radiation band energy has been obtained, the heat flux values $\Delta q_{l,k}$ for each band are found from equation (39). The total heat flux q_k (supplied or removed) is calculated as the sum of heat fluxes for the absorbing and non absorbing bands are:

$$q_k = \sum_{i=1}^L \sum_{j=1}^N \frac{\varepsilon_{i,k}}{1 - \varepsilon_{i,k}} \left[\frac{\delta_{ij}}{\varepsilon_{i,k}} - A_{kj}(l) \right] C_{lj} \tag{46}$$

Equation (46) relates to heat fluxes supplied to the surfaces at their given temperatures. It corresponds to equation (18) in the earlier formulation and provides a complete set of relations for the unknown q_k 's in terms of known T_j .

3.3. Band relations between surface temperature T and surface heating flux q

An interesting special case is when some surfaces have an imposed flux q specified together with respective temperatures T . The solution of such a problem is very complex and can be solved simultaneously for the distributions relating T and q . This is illustrated for the simple case of known temperatures T_j for surfaces $j = 1, 2, 3 \dots N-1$, and heat flux q_N for the surface N . As in the previous discussion for a grey open enclosure, there are two analytical steps that can now be taken:

- (1) the first step is to determine the temperature T_N of the surface N for which the imposed heat flux q_N is known. Equation (46) in the form shown above, and with k changed to N , takes the following form:

$$q_k = \sum_{i=1}^L \sum_{j=1}^{N-1} \frac{\varepsilon_{i,k}}{1 - \varepsilon_{i,k}} \left[\frac{\delta_{ij}}{\varepsilon_{i,k}} - A_{kj}(l) \right] C_{lj} \\ = \sum_{j=1}^L \frac{\varepsilon_{i,N}}{1 - \varepsilon_{i,N}} \left[\frac{1}{\varepsilon_{i,N}} - A_{NN}(l) \right] C_{l,N} \tag{47}$$

where

$$C_{l,N} = \varepsilon_{l,N} e_{ib,N} \Delta \lambda_l \tag{48}$$

The unknown temperature T_N in the form of the Planck function

$$e_{ib,N} = \frac{1}{\Delta \lambda_l} \int_{\Delta \lambda_l} e_{\lambda_c}(T_N) \Delta \lambda \tag{49}$$

provides the necessary value to guess the band width $\Delta \lambda_l$. The T_N value is used to select a new $\Delta \lambda_l$ and the process is repeated until T_N and $\Delta \lambda_l$ values converge and

- (2) after T_N is found the relations presented in Section 3.2 can be used to determine heat fluxes for all surfaces under consideration [8, 14].

3.4. Dissipation loss

The dissipation loss of an open enclosure filled with isothermal radiation gas (nongrey) is the sum of dissipation losses of all the bands. Some information on the directional nature of radiant dissipation losses from open system is given in [15], and takes the following form:

$$Q_{s,b} = \sum_{i=1}^L \sum_{j=1}^N \Delta Q_{0ij} \left(1 - \sum_{k=1}^N \bar{F}_{i,j-k} \right) \tag{50}$$

where

$$\Delta Q_{0ij} = A_i \Delta q_{0ij} \tag{51}$$

and the transfer band configuration factor $\bar{F}_{i,j-k}$ is specified by equation (41).

4. Analysis of two-surface open enclosure

In an earlier work [15] an open enclosure method was devised and applied to very general problems. The same basic approach will be applied here to the simple two-surface open enclosure of basic engineering interest. In this development it is not a variation of the net method which is discussed, but rather the way in which an open grey enclosure is treated within the context of the net method and transfer configuration factors. The solution to an open two-surface enclosure (Fig. 3(a)) is of engineering interest and is outlined in the following section.

4.1. Radiative heat exchange between two concave surfaces

Radiative heat exchange between two concave surfaces of an open system filled with isothermal radiation gas of a defined emission is equal to the difference between the heat energy supplied to surface A_1 and the dissipation loss of self emission $E_1(Q_{s,1})$ of this surface [2, 3]. The dissipation loss of self emission E_1 or eventually E_2 is determined from equation (35) on the assumption that $E_2 = 0$ or $E_1 = 0$.

According to the definition given in [1–3] the radiative heat exchange equals:

$$Q_{\tau,1-2} = Q_1 - Q_{s,1} \tag{52}$$

where

$$Q_1 = A_1[q_{0,1}(1 - \bar{F}_{1-1}) - q_{0,2}\bar{F}_{1-2}] \tag{53}$$

$$Q_{s,1} = \frac{\varepsilon_1 A_1 \sigma T_1^4}{Z} \{ [1 - (1 - \varepsilon_1)\bar{F}_{2-2}](1 - \bar{F}_{1-2} - \bar{F}_{1-1}) + (1 - \varepsilon_2)(1 - \bar{F}_{2-1} - \bar{F}_{2-2}) \} \tag{54}$$

Equation (54) was obtained from the dissipation loss equation of an open enclosure on the assumption that $E_2 = A_2 \varepsilon_2 \sigma T_2^4 = 0$. By substituting equations (53) and (54) into equation (52), the radiation heat exchange can be written as:

$$Q_{T,1-2} = \frac{\varepsilon_1 \varepsilon_2}{Z} A_1 \bar{F}_{1-2} \sigma (T_1^4 - T_2^4) \tag{55}$$

By introducing the abbreviation

$$\varepsilon_{T,1-2} = \frac{\varepsilon_1 \varepsilon_2}{Z} \bar{F}_{1-2} \tag{56}$$

equation (55) takes the following form

$$Q_{T,1-2} = \varepsilon_{T,1-2} A_1 \sigma (T_1^4 - T_2^4) \tag{57}$$

The quantity $\varepsilon_{T,1-2}$ is the substitute blackness degree for a two-surface open enclosure filled with radiation gas of a defined emission. It can be useful in the development of knowledge about radiative heat exchange in any systems filled with radiation gas or other optically active medium.

4.2. Steady processes—temperature of adiabatic surface

Although a number of different physical situations could be considered by theoretical analysis, each would have to be defined by its boundary interactions with an ambient.

However, a special use of this boundary condition is the Adiabatic Surface, that is a surface for which $Q_k = 0$. For example, in order to maintain the equilibrium of a two-surface open enclosure it is necessary to remove the heat output from surface A_2 . In matrix form, using the conventional notation of matrix algebra, the external heat transfer rate at Q_2 is obtained as [2, 15],

$$Q_2 = \frac{A_2}{Z} [\bar{F}_{2-1} \varepsilon_2 \varepsilon_1 \sigma T_1^4 - \varepsilon_2 \sigma T_2^4 \{ [1 - (1 - \varepsilon_1)\bar{F}_{2-1}\bar{F}_{1-2}] - \bar{F}_{1-1}(1 - \varepsilon_1)(1 - \bar{F}_{2-2}) - \bar{F}_{2-2} \}] \tag{58}$$

Now, by assuming that the surface A_2 is adiabatic ($Q_2 = 0$) the temperature as of an adiabatic surface T_a follows from equation (58) and the relationship between the temperatures T_1 and $T_2 = T_a$,

$$\frac{T_1}{T_2} = \frac{T_1}{T_a} = \sqrt[4]{\left(\frac{1}{\bar{F}_{2-1}\varepsilon_1} [1 - (1 - \varepsilon_1)\bar{F}_{1-2} - \bar{F}_{1-2}(1 - \varepsilon_1)(1 - \bar{F}_{2-2}) - \bar{F}_{2-2}] \right)} \tag{59}$$

The temperature $T_2 = T_a$ calculated from equation (59) is the maximum temperature which can be reached by the surface A_2 . An interesting consequence of these results is that the temperature of an adiabatic surface is not influenced by its radiative surface properties. Also

relation (59) yields the conclusion that the temperatures T_1 and T_2 need not be equal if the surface A_2 is adiabatic with respect to the environment. The only source of heat energy for surface A_2 is emission of surface A_1 through the gas mass, together with emission of the gas mass. According to Buraczewski [2] it may be stated that (i) the equilibrium temperature of an adiabatic surface $T_a < T_1$ and (ii) the equilibrium temperature of an adiabatic surface T_a is independent of radiative properties of surface A_2 , $T_a = f(T_1, \varepsilon, \bar{F})$. That is, the temperature T_a depends only on the emissive properties of radiation sources, i.e. surface A_1 , radiation gas and configuration of the enclosure.

4.3. Equilibrium temperature of radiation gas in a two-surface open enclosure

Making surface A_2 adiabatic with respect to ambient results in a rise in the temperature of (i) the surface A_2 to the adiabatic temperature T_a and (ii) the radiation gas to the temperature $T_{g,a}$. By analogy the latter can be called the radiation gas equilibrium temperature. The radiation gas equilibrium temperature $T_{g,a}$ can be derived from equation (12) which for a two-surface open enclosure can be evaluated as:

$$e_{g,b} = \frac{\varepsilon_1 \sigma A_1 T_1^4}{M_n} \times \frac{\left[(1 - \bar{F}_{2-2}) \sum_{z=1}^n F_{1-z} \varepsilon_{g,1-z} + \bar{F}_{1-2} \sum_{z=1}^n F_{2-z} \varepsilon_{g,2-z} \right]}{[1 - (1 - \varepsilon_1)\bar{F}_{2-1}\bar{F}_{1-2} - (1 - \varepsilon_1)\bar{F}_{1-1}(1 - \bar{F}_{2-2}) - \bar{F}_{2-2}]} \tag{60}$$

After substituting the radiation gas emission $e_{g,b} = \sigma T_{g,a}^4$ into equation (60) and abbreviating

$$M_n = \sum_{k=1}^n A_k \sum_{j=1}^n \varepsilon_{g,k-j} F_{k-j} \tag{61}$$

$$M_1 = \sum_{z=1}^n F_{1-z} \varepsilon_{g,1-z} \tag{62}$$

$$M_2 = \sum_{z=1}^n F_{2-z} \varepsilon_{g,2-z} \tag{63}$$

we obtain the relation for temperature ratio as

$$\frac{T_1}{T_g} = \frac{T_1}{T_{g,a}} = \frac{T_1}{T_a} \sqrt[4]{\frac{\bar{F}_{2-1} M_n}{A_1 [(1 - \bar{F}_{2-2}) M_1 + \bar{F}_{1-2} M_2]}} \tag{64}$$

From equation (23) it may be deduced that the temperature ratio which is established between surface A_1 and the radiation gas for adiabatic conditions of surface A_2 depends only on the shape and emissive properties of radiation gas [2, 15].

4.4. Unsteady process—emissivity as a function of the radiative property of the open enclosure

Equation (59) which describes the state of thermodynamic equilibrium of the open enclosure, allows the

determination experimentally of the emissivity ϵ of the adiabatic surface. As proved in [4] and [15], the time τ in which the adiabatic surface reaches equilibrium (from any initial state), depends on the surface emissivity ϵ . The assumption of surface A_2 (or A_1) being adiabatic with respect to the environment means that the heat input to the surface is accumulated and causes a rise in temperature up to an equilibrium state. In order to approach adiabatic conditions double-sided open enclosure was analysed as shown in Fig. 3(b). This condition has been achieved through dividing both the emitter (surface A_1) and receiver (surface A_2) into two equal parts and inverting one of them by 180°-on. In this way has a surface disappeared which has a heat loss, since the only means of heat transfer is by radiation. The energy absorbed by the adiabatic surface per unit time is

$$2Q_2 d\tau = mc_p dT = W dT \tag{65}$$

where m and c_p are the mass and specific heat of the adiabatic surface, respectively.

By substituting equation (58) in equation (65) we obtain

$$\epsilon_2 = \epsilon_a = \frac{1}{8R - \psi_\epsilon} \tag{66}$$

where

$$R = \frac{\sqrt[4]{F^3 T_1^3 A_2 \sigma \tau}}{W \left[\ln \frac{(T_a + \sqrt[4]{FT_1})(T_0 - \sqrt[4]{FT_1})}{(T_a - \sqrt[4]{FT_1})(T_0 + \sqrt[4]{FT_1})} + 2 \arctg \sqrt[4]{FT_1} \frac{T_a - T_0}{\sqrt[4]{FT_1^2 - T_a T_0}} \right]} \tag{67}$$

$$\psi_\epsilon = \frac{(1 - \epsilon_1) F_{1-2}^2}{1 - (1 - \epsilon_1) F_{1-2}^2} \tag{68}$$

and

$$F = \frac{\epsilon_1}{\left[\frac{1}{F_{1-2}^2} - (1 - \epsilon_1) F_{1-2} \right]} \tag{69}$$

In the above, T_0 is the initial temperature of surface A_2 . Equation (67) was obtained on the assumption that the open system under consideration is suspended inside a vacuum chamber.

5. Calorimetric emission measurement method

A survey of the literature over the past few years shows that calorimetric methods are continuing to receive much attention in preference to other approaches [8, 13, 14]. These are either steady-state or transient techniques, both requiring the measurements to be carried out under a vacuum of about 10^{-5} Torr. If only knowledge of the total, hemispherical emissivity of a surface is required,

this is most commonly determined by measuring the net radiative heat loss or gain of an isolated specimen.

5.1. Experimental setup and procedure

Experimental emissivity measurements have been carried out using the experimental setup shown in Fig. 4. By monitoring the temperature and heat losses the emissivity of the sample can be determined. The specimen is suspended between two-pseudo black emitters inside an evacuated test chamber, the walls of which are coated with a near-black material. The chamber walls are cooled, while the emitters are heated electrically, directly through the heating element. In the course of the experiment the sample and the emitters had been prepared earlier and placed opposite each other along the same horizontal axis, parallel and at equal corresponding distances. The sample and emitters were separated by two mirror shields. Temperatures of specimen, chamber wall and two emitters were monitored by thermocouples. As soon as an appropriate vacuum was obtained, the heating system of the emitters was turned on, and when the emitters and samples reached the desired temperature the shields were removed with a simultaneous temperature recording. The control system keeps the emitters at the same temperature by monitoring temperature with a differential thermocouple and taking corrective action whenever necessary. The temperatures of the sample were monitored until consistent successive measurements were obtained within the range of the instrument's measurement error. The resulting measurements were relative, therefore absolute values was obtained by calibrating the apparatus with a known standard in place-of the test sample and the experimental sample also was replaced by emitter-type material.

5.2. Experimental arrangements, results and uncertainty analysis

In this study, the hemispherical emissivity of five samples was measured at two different temperature of emitter (radiator) and two configuration factors F_{1-2} . Samples tested have been made as a circular slice with a diameter of 40 mm and thickness 2 mm (chamotte ~ 4.7 mm) with properly prepared surfaces in order to obtain stability of emission over the temperature range of measurement.

In the present experiment two geometries were investigated with $F_{1-2} = 0.388$ and 0.596 ($L = 10.5$ mm and 20.1 mm) for two emitter temperatures; $T_1 = 655$ and 728 K, respectively. For all experiments the initial temperature of samples was fixed and equal to 330.5 K. The radiator was a long copper cylinder of 40 mm in diameter, oxidised at 830 K and coated by candle soot.

The temperature distributions in the samples are shown in Figs. 5 and 6. By evaluating the model temperatures at time τ and the known properties of the model

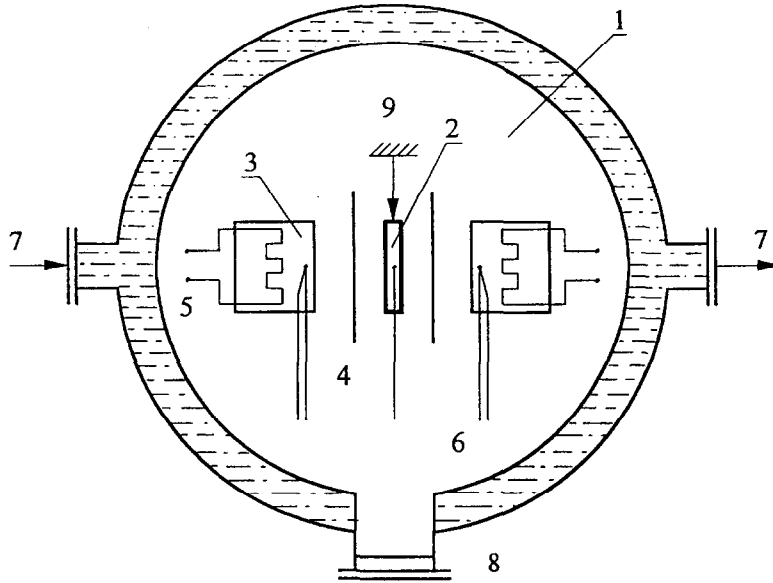


Fig. 4. Schematic to the experimental setup for calorimetric emission measurements 1. stainless steel vacuum jacket, 2. sample, 3. heating radiator, 4. mirror radiation shields, 5. power leads, 6. thermocouple leads, 7. inlet and outlet coolant, 8. sample viewing port, 9. blade sample holder.

substrate, ϵ_a may be evaluated. Table 1 shows experimental results of ϵ_a obtained from equation (66) together with the corresponding uncertainties. In order to estimate the discrepancies of the present method a calibration test was performed by using a pseudo-blackbody sample mounted in a measurement specimen. From this, the deviation from the value ϵ_a obtained from measurement caused by discrepancies, was found to be 0.022. Finally, emissivity value of samples will be found by the following formula :

$$\epsilon = (\epsilon_a + 0.022) \pm \frac{\Delta\epsilon_a}{\epsilon_a} \tag{70}$$

where $\Delta\epsilon_a/\epsilon_a$ is the experimental uncertainty calculated using a root-sum-square (RSS) combination [9].

6. Calculation of examples using the above procedures

Further to aid understanding of the implications of the present method two examples are now presented. The first one is an analytically defined example to acquaint the reader with the use of the analytical relations for an open grey enclosure filled with isothermal radiation gas. In the calculation of the second example, the band radiation heat transfer is explained, and it provides a bridge between theory and practical application.

Example 1.

Problem: Two perfectly black, parallel, infinitely long

plates of width $W = 1$ m are separated by $D = 1$ m (Fig. 7). The space between the plates is filled with carbon dioxide at $P = 0.101$ MPa and $T_g = 1000$ K. If plates 1 and 2 maintained at 2000 and at 500 K, respectively, determine :

- (1) the heat flux q_2 that must be supplied to plate 2 to maintain its temperature,
- (2) the equilibrium gas temperature $T_{g,e}$ if the temperature of the surroundings is low at $T_e \ll 500$ K.

Solution: To simplify the treatment, which is discussed in [4], the concept is used of considering the entire wavelength region as a single band or grey gas. To determine τ_{j-k} and ϵ_{gt-k} the geometric-mean beam length is used, this technique being given in Siegel and Howell [14]. For the present case the following values of the emissivity and transmissivity and configuration factors are obtained :

$$\begin{aligned} \epsilon_{g,2-3} = \epsilon_{g,1-2} = 0.22 & \quad F_{2-3} = F_{2-4} = F_{1-3} = F_{1-4} = 0.293 \\ \epsilon_{g,2-1} = \epsilon_{g,2-4} = 0.19 & \quad F_{3-2} = F_{4-2} = F_{3-1} = F_{4-1} = 0.293 \\ \epsilon_{g,1-3} = \epsilon_{g,1-4} = 0.19 & \quad F_{1-2} = F_{2-1} = 0.414 \\ \epsilon_{g,3-4} = \epsilon_{g,4-3} = 0.22 & \quad F_{1-1} = F_{2-2} = 0 \\ \epsilon_1 = \epsilon_2 = 1 & \\ \tau_{2-1} = 0.87. & \end{aligned}$$

The heat flux q_2 supplied from plate A_2 necessary to maintain the assumed equilibrium state is calculated from equation (18) as :

$$q_2 = \sigma T_1^4 \bar{F}_{2-1} - \sigma T_2^4 (1 - \bar{F}_{2-2}). \tag{71}$$

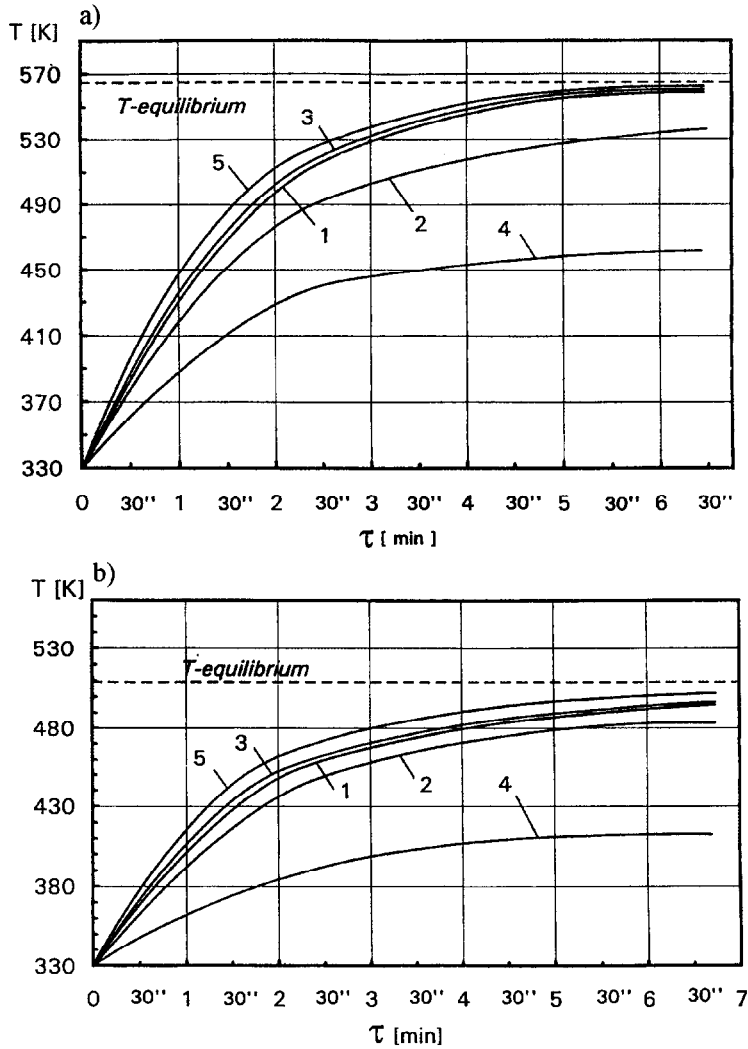


Fig. 5. Temperature dependence of heating sample by radiation in the transient calorimetric technique $T_1 = 655 \text{ K}$, $T_0 = 330.5 \text{ K}$, (a) $F_{1-2} = 0.596$, (b) $F_{1-2} = 0.388$ (1. oxide coated copper, 2. oxide copper, 3. nickelin, 4. cast iron, 5. chamotte).

The transfer configuration factors \bar{F}_{2-1} and \bar{F}_{2-2} remain to be found. From equation (20) the \bar{F}_{2-1} , is given by:

$$\bar{F}_{2-1} = \left[F_{2-1}\tau_{2-1} + A_1 \frac{\sum_{z=1}^n F_{1-z}\epsilon_{g,1-z}}{\sum_{j=1}^n F_{2-j}\epsilon_{g,2-j}} \right] / \left[F_{2-1}\tau_{2-1} + \frac{B}{M_n} \right] \quad (72)$$

where

$$B = A_1 \{ (F_{2-1}\epsilon_{g,2-1} + F_{2-2}\epsilon_{g,2-2} + F_{2-3}\epsilon_{g,2-3} + F_{2-4}\epsilon_{g,2-4}) \cdot (F_{1-1}\epsilon_{g,1-1} + F_{1-2}\epsilon_{g,1-2} + F_{1-3}\epsilon_{g,1-3} + F_{1-4}\epsilon_{g,1-4}) \} \\ = A_1 (0.419 \cdot 0.22 + 0.293 \cdot 0.19 + 0.293 \cdot 0.19)^2 \quad (73)$$

$$M_n = A_1 (F_{1-1}\epsilon_{g,1-1} + F_{1-2}\epsilon_{g,1-2} + F_{1-3}\epsilon_{g,1-3} + F_{1-4}\epsilon_{g,1-4}) \\ + A_4 (F_{4-1}\epsilon_{g,4-1} + F_{4-2}\epsilon_{g,4-2} + F_{4-3}\epsilon_{g,4-3} + F_{4-4}\epsilon_{g,4-4}) \\ = 4 \cdot A_1 (0.414 \cdot 0.22 + 0.293 \cdot 0.19 + 0.293 \cdot 0.19). \quad (74)$$

Equation (73) is obtained on the assumption that $A_1 = A_2 = A_3 = A_4$. In this instance \bar{F}_{2-1} is found by substitution of equations (74) and (73) into (72),

$$\bar{F}_{2-1} = [0.414 \cdot 0.87 + 1/4 \cdot (0.414 \cdot 0.22 + 2 \cdot 0.19 \cdot 0.293)] = 0.41.$$

Using the same equation (20) the value of transfer configuration factor $\bar{F}_{2-2} = 0.05$ and then

$$q_2 = 5.7310 \cdot 10^{-12} [2000^4 \cdot 0.41 - 500^4 \cdot 0.95] = 37.25 \text{ W/cm}^2.$$

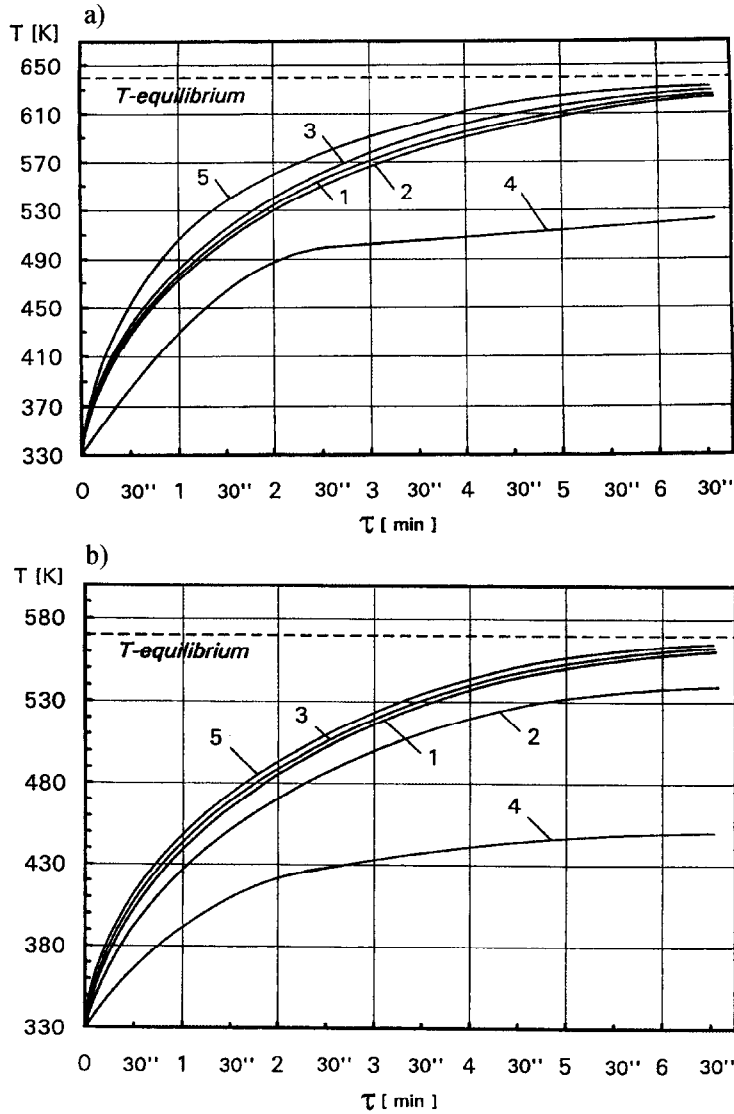


Fig. 6. Temperature dependence of heating sample by radiation in the transient calorimetric technique $T_1 = 728$ K, $T_0 = 330.5$ K, (a) $F_{1-2} = 0.596$, (b) $F_{1-2} = 0.388$ (1. oxide coated copper, 2. oxide copper, 3. nickelin, 4. cast iron, 5. chamotte).

The solution is now almost complete. The largest contribution to q_2 is by energy-leaving surface 1 and being absorbed by surface 2. Emission from the gas to surface 3 cannot be taken to be negligible. Consequently, the gas temperature T_g is established at a somewhat higher level accordingly to the assumed boundary conditions. The real gas temperature can be calculated from equation (12). For the case under consideration the radiation gas temperature is given by the following expression [15]:

$$T_g = \sqrt[4]{\frac{T_1^4 + T_2^4}{4}} = \sqrt[4]{\frac{500^4 + 2000^4}{4}} \cong 1415 \text{ K.} \quad (75)$$

The above calculations give a value for q_2 (the amount

of heat which must be carried away from surface A_2 in order to maintain the assumed equilibrium state) of 37.25 W/cm^2 . The comparable calculation made by Siegel and Howell [14] gave $q_2 = 33.8 \text{ W/cm}^2$. The difference in the result ($\Delta q = 3.45 \text{ W/cm}^2$ i.e. about 10%) is caused by the arbitrary level taken for the radiation gas temperature.

Example 2.

Problem: Parallel nongrey plates (made of titanium oxide) are at a distance $D = 0.0254$ m from each other. The temperatures of the surfaces are constant and equal: $T_1 = 1100$ K and $T_2 = 556$ K, respectively. Pure CO_2 gas at $P = 1.01$ MPa pressure and $T = 556$ K fills the space

Table 1
Total normal emissivity of various surface and experimental uncertainty

Material	ϵ at 700 K [19] [28]	T_1 [K]	$F_{1-2} = 0.596$		$F_{1-2} = 0.388$	
			ϵ	$\Delta\epsilon_a/\epsilon_a$	ϵ	$\Delta\epsilon_a/\epsilon_a$
1. Oxide copper coated by candle soot	0.95	655	0.946	$\pm 2\%$	0.947	$\pm 2.5\%$
		728	0.944	± 3	0.938	$\pm 4\%$
2. Oxide copper at 780 K (covered with thick oxide layer)	0.63	655	0.623	± 4.5	0.625	$\pm 5.5\%$
		728	0.625	± 5	0.637	$\pm 6.5\%$
3. Nickelin grey oxidised	0.27	655	0.253	± 6.5	0.275	$\pm 7\%$
		728	0.299	± 7.5	0.291	$\pm 8\%$
4. Cast iron, rough, strongly oxidised	0.95	655	0.979	± 2	0.945	$\pm 2.5\%$
		728	0.945	± 3	0.956	$\pm 4\%$
5. Chamotte, rough and white-grey	0.98	655	0.982	± 2	0.985	$\pm 2.5\%$
		728	0.953	± 3	0.970	± 4

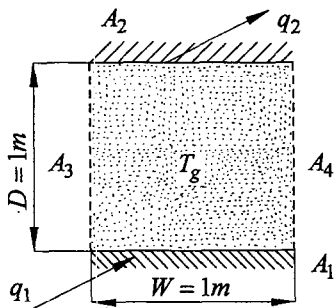


Fig. 7. Isothermal carbon dioxide contained between black plates.

between the g plates. The plate hemispherical spectral emissivity as a function of wave number is approximated and shown in Table 2.

In the table it is assumed that only $\lambda = 15, 10.4, 9.3, 4.3,$ and $2.7 \mu\text{m}$ bands cause significant attenuation in the gas. Compute the total heat flux carried away from the plate 2.

Solution: To obtain the heat flux carried away from

Table 2
Values of titanium oxide hemispherical spectral emissivity

ν [cm^{-1}]	ϵ_1	ν [cm^{-1}]	ϵ_1
0–500	0.37	1150–2200	0.45
500–750	0.26	2200–2500	0.65
750–850	0.32	2500–3600	0.62
850–1000	0.37	3600–3750	0.69
1000–1150	0.46	3750– ∞	0.73

A_2 equation (46) must be evaluated. This results in the relations:

$$\Delta q_{l,2} = \Delta q_{0l,1} \bar{F}_{l,2-1} - \Delta q_{0l,2} (1 - \bar{F}_{l,2-2}). \quad (76)$$

The band effective energy fluxes $\Delta q_{0l,1}$ and $\Delta q_{0l,2}$ in equation (76) are obtained from equation (58) as

$$\Delta q_{0l,1} = \frac{Z_{l,1}}{Z_l}; \quad \Delta q_{0l,2} = \frac{Z_{l,2}}{Z_l} \quad (77)$$

and the coefficients $Z_b, Z_{l,1}$ and $Z_{l,2}$ equal, respectively:

$$Z_{l,1} = \begin{bmatrix} \epsilon_{l,1} e_{lb,1} \Delta \lambda_l & -(1 - \epsilon_{l,1}) \bar{F}_{l,1-2} \\ \epsilon_{l,2} e_{lb,2} \Delta \lambda_l & -(1 - \epsilon_{l,2}) \bar{F}_{l,2-2} \end{bmatrix} \quad (78)$$

$$Z_{l,2} = \begin{bmatrix} 1 - (1 - \epsilon_{l,1}) \bar{F}_{l,1-1} & -(1 - \epsilon_{l,1}) \bar{F}_{l,1-2} \\ -(1 - \epsilon_{l,2}) \bar{F}_{l,2-1} & 1 - (1 - \epsilon_{l,2}) \bar{F}_{l,2-2} \end{bmatrix} \quad (79)$$

$$Z_l = \begin{bmatrix} 1 - (1 - \epsilon_{l,1}) \bar{F}_{l,1-1} & -(1 - \epsilon_{l,1}) \bar{F}_{l,1-2} \\ -(1 - \epsilon_{l,2}) \bar{F}_{l,2-1} & 1 - (1 - \epsilon_{l,2}) \bar{F}_{l,2-2} \end{bmatrix} \quad (80)$$

If we now assumed that the emission coefficients of the plate A_1 and A_2 are equal and independent of temperature $\Delta q_{l,2}, \bar{F}_{l,1-1}$ and $\bar{F}_{l,1-2}$ may be written as

$$\Delta q_{l,1} = \frac{[\epsilon_l \bar{F}_{l,1-2} - \epsilon_l (1 - \epsilon_l) \bar{F}_{l,1-2} \bar{F}_{l,1-1} - \epsilon_l (1 - \epsilon_l) \bar{F}_{l,1-2}^2] \times [e_{lb,1} - e_{lb,2}] \Delta \lambda_l}{[1 - (1 - \epsilon_l) \bar{F}_{l,1-1}]^2 - [(1 - \epsilon_l) \bar{F}_{l,1-2}]^2} \quad (81)$$

$$\bar{F}_{l,1-1} = \frac{\epsilon_{l,1-2}}{2} = \frac{A_l}{2\Delta \nu_l} \quad (82)$$

$$\bar{F}_{l,1-2} = 1 - \bar{F}_{l,1-1} = 1 - \frac{\epsilon_{l,1-2}}{2} = 1 - \frac{A_l}{2\Delta \nu_l} \quad (83)$$

where A_l is the integral bandwidth that includes the integrated path-length variation for a parallel plate geometry.

Values of A_l are now computed from the exponential wide-band correlations proposed by Edwards [5] on the assumption that the geometric mean radiation path

length of the gas in shape of unlimited plate $\bar{S} = 1.8 D$. The wavenumber spans Δv_i are computed from Table 13-3 of Siegel and Howell [14]. For the nonabsorbing regions the $[e_{ib,1} - e_{ib,2}] \Delta \lambda_i$ are computed using F_{0-2T} , that is, $E_{ib,1}(T_1) \Delta \lambda_i = F_{\lambda_1 T_1 - \lambda_2 T_1} e_{ib}(T_1)$, where λ_1 and λ_2 corresponds to the wavenumber limits of the band. For the absorbing regions: $e_{ib,1}(T_1) \Delta \lambda_i = [e_b(T_1)]_{\text{bandcentre}} \Delta \lambda_i$. When the band correlations give $A_i > \Delta v_i$ then $A_i / \Delta v_i = 1.0$ are used as physically A_i cannot exceed Δv_i . The required variables and their values are shown in Table 3. In the last column of Table 3 the values of heat fluxes calculated by Edwards and Nelson [6] and Siegel and Howell [14] are presented. The calculated heat flux q_2 outgoing from plate A_2 is 3326 W/m² greater (about 10%) than that computed by Edwards and Nelson [6] and Siegel and Howell [14]. Edwards and Nelson performed their calculation based on the network method of Oppenheim [11]. The differences (shown in Table 3) in the computed values for q_2 results from the greater simplifications involved in the method used by the above authors relative to current method. According to the calculation model presented in [6] and [14] for black bands ($\tau_{i,1-2} = 0$), where the gas temperature is equal to the temperature of plate A_2 , $\Delta q_{i,2}$ becomes 0 (position 7 and 9 in Table 3). However even with the suppositions that the gas temperature T_g equals the temperature of the plate A_2 ($T_g = T_2$) and the band transmissivity $\tau_{i,1-2} = 0$ according to the new method here and an extensive discussion given by Buraczewski [2] emission from the surface A_1 into surface A_2 through the gas will occur. A calculation model which yields a band heat flux supplied $\Delta q_{i,2} = 0$ (black bands) in the case when $T_1 = 1110$ K,

means that the gas temperature T_g equals the temperature of the plate 2 $T_2 = T_g = 556$ K, this does not seem to be correct. The temperature gradient existing between plate 2 and the gas causes an emission transfer from this surface to the gas. In order to keep the open enclosure in the assumed state of equilibrium the energy absorbed by the gas must be transferred to the surface A_2 and be removed as the outgoing heat flux $\Delta q_{i,2}$ or, in the opposite situation, the gas temperature must increase until equilibrium is reached [2, 15].

The present model, however, does not use the gas temperature T_g directly because it is a function of the incoming, outgoing and scattering heat fluxes. Also the new method describes more correctly the situation if $\tau_{i,1-2} = 0$ in which emission from plate 1 to plate 2 occurs through the gas. Gas of transmissivity $\tau_{i,1-2} = 0$ acts as a shield placed between two surfaces, in such a case the shield temperature would rise to a value between T_1 and T_2 .

Hence, in order to perform the correct calculation, the gas temperature corresponding to the state of thermodynamic equilibrium has to be found. This may be done from equations (13) and (17) expressed in the following form :

$$\sum_{l=1}^L \sum_{k=1}^n A_k e_{ib} \Delta \lambda_i \sum_{j=1}^n \epsilon_{i,k-j} F_{k-j} = \sum_{l=1}^L \sum_{j=1}^N A_j \Delta q_{0lj} \sum_{z=1}^n F_{j-z} \epsilon_{lj-z} \tag{84}$$

Equation (84) is solved by iteration because the effective energy fluxes Δq_{0ij} and quantity e_{ib} are functions of the gas temperature which is being sought.

For the case under consideration it was found that the

Table 3
Quantities and results for band radiation between parallel plates

No.	Band ν [m ⁻¹]	ϵ_1	Band width $\Delta \nu_i$ [m ⁻¹]	$[e_{ib,1} - e_{ib,2}] \Delta \lambda_i$ [W/m ²]	A_i [W/m ²]	$\Delta q_{i,2}$ [W/m ²]	$\Delta q_{i,2}$ [W/m ²] [8] [28]
1	0-55 525	0.37	55 525	790	0	179.5*	180.0*
2	55 525-77 874 (15 μm)	0.26	22 349	1300	19 890	180.0	9.5
3	77 874-84 900	0.32	7 026	500	0	95.0*	95.0*
4	84 900-101 300 (10.4 μm)	0.37	16 400	1860	960	410.0	369.0
5	101 300-114 100 (9.4 μm)	0.46	12 800	1520	960	449.0	394
6	114 100-215 700	0.45	101 600	19 910	0	5780.0*	6170.0*
7	215 700-241 000 (4.3 μm)	0.65	25 300	6020	22 520	2090.0	0
8	24 100-351 800	0.61	110 800	23 130	0	10 150.0*	10 200*
9	351 800-380 200 (2.7 μm)	0.69	28 400	4250	24 960	1583.0	0
10	380 200- ∞	0.73		22 300	0	12 818.0*	13 000
						$q_2 = \Sigma \Delta q_{i,2} = 33 743.5$	30 417.5

temperature for thermodynamic equilibrium is $T_{g,e} = 940$ K. The calculation was performed for values $\tau_{l,1-2}$ and $\varepsilon_{l,1-2}$ as for $T_g = 556$ K. More accurate results can be obtained if in subsequent trial the influence of temperature g on $\tau_{l,1-2}$ and $\varepsilon_{l,1-2}$ and the band width is included.

7. Conclusions

In this paper a new engineering method was developed and solution presented for analysing radiative energy exchange within an open enclosure filled with an isothermal radiation gas having black, diffuse-grey or spectral property variations. The relation for exchange between multi-surface open enclosure is then applied to two surfaces, each at a different uniform temperature and boundary conditions. Two illustrative examples and experimental results are presented.

The principal novel feature of new method is the introduction of the transfer configuration factor (here called \bar{F}) applicable to practical engineering situations and simplify enclosure theory. The method may readily be utilised in a computational procedure. The grey-diffuse and spectral net-radiation model used in the present study is also evaluated.

Comparison of this model with previous analyses are cited showing favourable agreement.

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